Lecture Series – 4

Solving Algebraic Equations
DC Circuit Analysis

by

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Sets of Linear Equations

- $x_1+2x_2+3x_3 = 366$
- $4x_1+5x_2+6x_3 = 804$
- $7x_1+8x_2+9x_3 = 351$
- In matrix form

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 366 \\ 804 \\ 351 \end{bmatrix}$$

- This is in the form

$$A * x = y$$

$$\text{Hence} \quad x = A^{-1} y$$

Solving Linear Equations

- The problem have a solution if rank of A and rank of augmented matrix, ([A y]), are equal.
- *rank (A)*
- *rank ([A y])*

rank of a matrix A is the maximum number of linearly independent row vectors of A

- Then test condition number of A, which should be a small number (close to 1 is better)

(the condition number associated with the linear equation Ax = b gives a bound on how inaccurate the solution x will be after approximation)

- *cond (A)*
- Now the solution is
- *x= inv(A)*y* or *x = A\y*

D C Loop Analysis

$$Z_{11}I_1 + Z_{12}\, I_2 + Z_{13}\, I_3 + \quad \ldots \quad Z_{1n}\, I_n = \sum V_1$$

$$Z_{21}\, I_1 + Z_{22}\, I_2 + Z_{23}\, I_3 + \quad \ldots \quad Z_{2n}\, I_n = \sum V_2$$

$$Z_{n1}\, I_1 + Z_{n2}\, I_2 + Z_{n3}\, I_3 + \quad \ldots \quad Z_{nn}\, I_n = \sum V_n$$

$I_1, I_2, \ldots I_n$ are the unknown currents for meshes 1 through n.

$Z_{11}, Z_{22}, \ldots, Z_{nn}$ are the impedance for each mesh through which individual current flows.

Z_{ij}, $j \# i$ denote mutual impedance.

$\sum V_x$ is the algebraic sum of the voltage sources in mesh x.

D C Loop Analysis ...

$$[Z][I] = [V]$$

where

$$Z = \begin{bmatrix} Z_{11} & Z_{12} & Z_{13} & \dots & Z_{1n} \\ Z_{21} & Z_{22} & Z_{23} & \dots & Z_{2n} \\ Z_{31} & Z_{32} & Z_{33} & \dots & Z_{3n} \\ .. & .. & . & \dots & .. \\ Z_{n1} & Z_{n2} & Z_{n3} & \dots & Z_{nn} \end{bmatrix}$$

$$I = \begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ . \\ I_n \end{bmatrix}$$

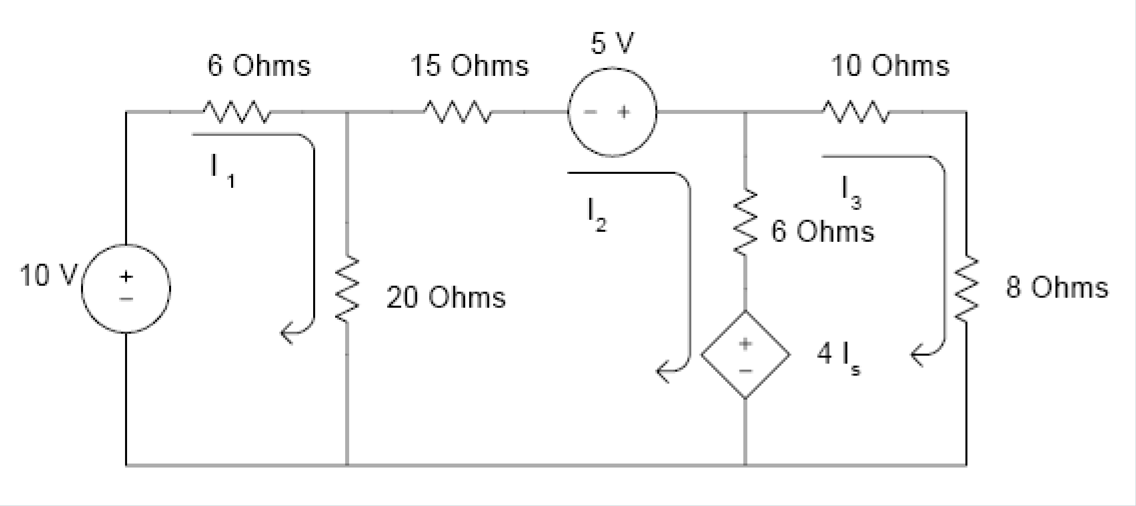
$$V = \begin{bmatrix} \sum V_1 \\ \sum V_2 \\ \sum V_3 \\ .. \\ \sum V_n \end{bmatrix}$$

$$[I] = [Z]^{-1}[V]$$

In MATLAB, we can compute [I] by using the command

$$I = inv(Z) * V \qquad I = Z \backslash V$$

Example Circuit



For loop 1,

$$-10 + 6I_1 + 20(I_1 - I_2) = 0$$

$$26I_1 - 20I_2 = 10$$

For loop 2,

$$15I_2 - 5 + 6(I_2 - I_3) + 4I_S + 20(I_2 - I_1) = 0$$

$$-16I_1 + 41I_2 - 6_3I = 5$$

For loop 3,

$$10I_3 + 8I_3 - 4I_S + 6(I_3 - I_2) = 0$$

$$-4I_1 - 6I_2 + 24I_3 = 0$$

Example Circuit .....

- In matrix form:

$$\begin{bmatrix} 26 & -20 & 0 \\ -16 & 41 & -6 \\ -4 & -6 & 24 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 10 \\ 5 \\ 0 \end{bmatrix}$$

- Matlab Script:

  - Z = [26 -20 0; -16 41 -6; -4 -6 24]
  - V = [10; 5; 0]
  - Rank (Z)
  - Rank ([Z V])
  - Cond (A)
  - I = Z\V

Thanks

Do the exercises

Questions ??

Mid Lab: Week 8`