## Lecture Series - 3

## Matrices and Arrays in MATLAB



Shameer Koya

## MATLAB Matrices

- MATLAB treats all variables as matrices. For our purposes a matrix can be thought of as an array, in fact, that is how it is stored.
- Vectors are special forms of matrices and contain only one row OR one column.
- Scalars are matrices with only one row AND one column


## Arrays

A rectangular arrangement of numbers is called an array.


This is a 4-by-2 array. It has 4 rows, and 2 columns.

The $(3,1)$ entry is the entry in the $3^{\text {rd }}$ row, and $1^{\text {st }}$ column

Later, we will have rectangular arrangement of more general objects. Those will also be called arrays.

## Entering a Matrix in MATLAB

-Enter an explicit list of elements
-Load matrices from external data files
-Using built-in functions
-Using own functions in M-files
-A matrix can be created in MATLAB as follows (note the commas AND semicolons):
» matrix $=\left[\begin{array}{llllllllll}1 & 2 & 3 ; 4 & 5 & 7 & 8 & 9\end{array}\right]$

## Entering a Matrix in MATLAB ....

->> A = [2-3 5; -1 4 5]

- $\mathrm{A}=$
- $2 \begin{array}{lll}-3 & 5\end{array}$
- -145
- $\gg x=\left[\begin{array}{lll}1 & 4 & 7\end{array}\right]$
-X $=$
- 147
$\rightarrow>x=[1 ; 4 ; 7]$
- $\mathrm{X}=$
- 1
- 4

$$
\begin{gathered}
\mathbf{A}=\left[\begin{array}{ccc}
2 & -3 & 5 \\
-1 & 4 & 6
\end{array}\right] \\
\mathbf{x}=\left[\begin{array}{lll}
1 & 4 & 7
\end{array}\right] \\
\mathbf{x}=\left[\begin{array}{l}
1 \\
4 \\
7
\end{array}\right]
\end{gathered}
$$

## Generating Matrices Using Built-in Functions

## Matrix with ZEROS:

zeros (r, c)
Matrix with ONES:
ones ( $\mathrm{r}, \mathrm{c}$ )

## IDENTITY Matrix:

eye (r, c)

## Random Matrix:

$\operatorname{rand}(\mathrm{r}, \mathrm{c})$

Examples

$$
\begin{aligned}
& \gg A=\operatorname{ones}(3,2) ; \\
& \gg B=\operatorname{zeros}(3,4) ; \\
& \gg C=\operatorname{rand}(2,5) ; \\
& \gg C=\text { magic(4); - Durer's matrix }
\end{aligned}
$$

## Built-in Functions to Handle Matrices / Arrays

, $\operatorname{Sum}()$ - sum of elements in a column
, '-Transpose of a matrix
, Diag() - diagonal elements of matrix

- Size() - size (dimensions) of matrix
- size(A)
- ans =

24
size of A: 2 rows, 4 columns
, $\operatorname{Det}()$ - determinant of a matrix.
, $\operatorname{Inv}()$ - inverse of a matrix

## The Colon operator

- A colon notation is an important shortcut, used when producing row vectors

```
>> 2:5
ans =
2345
>> -2:3
ans =
-2 -1 O 1 2 3
format - first:step:last
>> 0.2:0.5:2.4
ans =
0.2000 0.7000 1.2000 1.7000 2.2000
>> -3:3:10
ans =
-30369
>> 1.5:-0.5:-0.5 (negative step is also possible)
ans =
1.5000 1.0000 0.5000 0 -0.5000
```


## Subscripts and Extracting a Sub-Matrix

- $A(i, j)$ - element $i^{\text {th }}$ row, $j^{\text {th }}$ column
- $A(k)$
- A portion of a matrix can be extracted and stored in a smaller matrix by specifying the names of both matrices and the rows and columns to extract. The syntax is:

$$
\mathrm{A} 1=\mathrm{A}(\mathrm{r} 1: \mathrm{r} 2, \mathrm{c} 1: \mathrm{c} 2)
$$

where r1 and r2 specify the beginning and ending rows and c 1 and c2 specify the beginning and ending columns to be extracted to make the new matrix.

## Extracting a Sub-Matrix ....

- >> A(3,:) \% extract the 3rd row of A
- ans = 789
- >> A(:,2) \% extract the 2nd column of A
- ans =

2
5
8

- >> A([1,3],1:2) \% extract a part of A
- ans =

12
78

$$
» A(4,1)
$$

??? Index exceeds matrix dimensions.

## Adding And Deleting Elements

- Indexing can be used to add and delete elements from a matrix.
- $\gg \mathrm{A}(5,2)=5 \quad \%$ assign 5 to the position (5,2); the uninitialized elements become zeros
- $\mathrm{A}=$

123
458
789
0 o
O 50

- $\gg \mathrm{A}(4,:)=[2,1,2] ; \%$ assign vector $[2,1,2]$ to the 4 th row of A
- $\gg \mathrm{A}(5,[1,3])=[4,4] ; \%$ assign: $\mathrm{A}(5,1)=4$ and $\mathrm{A}(5,3)=4$


## Adding And Deleting Elements ...

- $\mathrm{A}(1,2)=5-$ will replace the element in position $(1,2)$ with 5
- $A(4,:)=[]-$ will delete $4^{\text {th }}$ row
- $A(:, 3)=[]-$ will delete $3^{\text {rd }}$ column
- $A(1,2)=[]-$ error
- Can't delete single element in a row or column.
- $\mathrm{A}(2: 2: 6)=[]$
- ans $=175369$
- ................ How?


## Operators

| + |
| :--- |
| - |
| $*$ |
| * |
| $\vdots$ |

## addition

subtraction
multiplication
power
transpose
left division
right division

## Operations on Matrices

```
>> B = [1 -1 3; 4 0 7}
B =
\begin{tabular}{rrr}
1 & -1 & 3 \\
4 & 0 & 7
\end{tabular}
>> B2 = [1 2; 5 1; 5 6];
> B = B + B2'
B =
248
613
>> B-2 % subtract 2 from all elements of B
ans =
\begin{tabular}{rrr}
0 & 2 & 6 \\
4 & -1 & 11
\end{tabular}
>> ans = B./4 % divide all elements of the matrix B by 4
ans =
\begin{tabular}{lll}
0.5000 & 1.0000 & 2.0000 \\
1.5000 & 0.2500 & 3.2500
\end{tabular}
    % this is not possible
??? Error using ==> /
Matrix dimensions must agree.
```


## Operations on Matrices

>> b = [lll $\left.1 \begin{array}{lll}1 & 3 & -2\end{array}\right] ;$
$\gg B=\left[\begin{array}{cccccc}1 & -1 & 3 ; & 4 & 0 & 7\end{array}\right]$
B =

| 1 | -1 | 3 |
| :--- | :--- | :--- |

$\gg \mathrm{b} * \mathrm{~B} \quad \%$ not possible: b is 1 -by-3 and B is 2-by-3
??? Error using ==> *
Inner matrix dimensions must agree.
> $\mathrm{b} * \mathrm{~B}^{\prime}$
ans =
-8 -10
>> $C=\left[\begin{array}{llllll}1 & -1 & 4 ; 7 & 0 & -1\end{array}\right]$;
>> B .* C
\% multiply element-by-element
ans =

| 2 | -4 | 32 |
| :--- | :--- | :--- |
| 42 | 0 | -13 |

0 -13
>> ans.^3-2
ans =
$\begin{array}{rrr}6 & -66 & 32766 \\ 74086 & -2 & -2199\end{array}$

## Operations on Matrices - Summary

| Command <br> $\mathrm{C}=\mathrm{A}+\mathrm{B}$ | Result <br> sum of two matrices |
| :--- | :--- |
| $\mathrm{C}=\mathrm{A}-\mathrm{B}$ | subtraction of two matrices |
| $\mathrm{C}=\mathrm{A} * \mathrm{~B}$ | multiplication of two matrices |
| $\mathrm{C}=\mathrm{A} \cdot * \mathrm{~B}$ | 'element-by-element' multiplication $(A$ and $B$ are of equal size $)$ |
| $\mathrm{C}=\mathrm{A} \wedge \mathrm{k}$ | power of a matrix $\left(k \in Z ;\right.$ can also be used for $\left.A^{-1}\right)$ |
| $\mathrm{C}=\mathrm{A} \cdot \wedge \mathrm{k}$ | 'element-by-element' power of a matrix |
| $\mathrm{C}=\mathrm{A}{ }^{\prime}$ | the transposed of a matrix; $A^{T}$ |
| $\mathrm{C}=\mathrm{A} . / \mathrm{B}$ | 'element-by-element' division $(A$ and $B$ are of equal size $)$ |
| $\mathrm{X}=\mathrm{A} \backslash \mathrm{B}$ | finds the solution in the least squares sense to the system of equations $A X=B$ |
| $\mathrm{X}=\mathrm{B} / \mathrm{A}$ | finds the solution of $X A=B$, analogous to the previous command |
|  |  |
| $\mathrm{A}=$ eye $(\mathrm{n})$ | $A$ is an $n \times n$ identity matrix |
| $\mathrm{A}=\mathrm{zeros}(\mathrm{n}, \mathrm{m})$ | $A$ is an $n \times m$ matrix with zeros (default $m=n)$ |
| $\mathrm{A}=$ ones $(\mathrm{n}, \mathrm{m})$ | $A$ is an $n \times m$ matrix with ones $($ default $m=n)$ |
| $\mathrm{A}=\operatorname{diag}(\mathrm{v})$ | gives a diagonal matrix with the elements $v_{1}, v_{2}, \ldots, v_{n}$ on the diagonal |

## Matrix Addition and Subtraction

- Matrix addition and subtraction with MATLAB are achieved in the same manner as with scalars provided that the matrices have the same size. Typical expressions are shown below.
- $\gg \mathrm{C}=\mathrm{A}+\mathrm{B}$
->> D $=\mathrm{A}-\mathrm{B}$


## Matrix Multiplication

Matrix multiplication with MATLAB is achieved in the same manner as with scalars provided that the number of columns of the first matrix is equal to the number of rows of the second matrix. A typical expression is
$\gg E=A * B$

## Array Multiplication

-There is another form of multiplication of matrices in which it is desired to multiply corresponding elements in a fashion similar to that of addition and subtraction. This operation arises frequently with MATLAB, and we will hereafter refer to the process as the array product to distinguish it from the standard matrix multiplication form.

## MATLAB Array Multiplication

-To form an array product in MATLAB, a period must be placed after the first variable. The operation is commutative. The following two operations produce the same result.
->> $\mathrm{F}=\mathrm{A} .{ }^{*} \mathrm{~B}$
-> $\mathrm{F}=\mathrm{B} .{ }^{*} \mathrm{~A}$

## MATLAB Array Multiplication Continuation

If there are more than two matrices for which array multiplication is desired, the periods should follow all but the last one in the expression; e. g., A.*B.*C in the case of three matrices. Alternately, nesting can be used; e.g. (A.*B).*C for the case of three matrices.

## MATLAB Array Multiplication Conti@lution

-The array multiplication concept arises in any operation in which the command could be "confused" for a standard matrix operation. For example, suppose it is desired to form a matrix B from a matrix A by raising each element of A to the 3rd power, The MATLAB command is
->>B $=A .{ }^{\wedge} 3$

## Multi-dimensional Array

- Arrays can have more than two dimensions
- $A=\left[\begin{array}{ll}12 ; & 4\end{array}\right]$
- You can add the $3^{\text {rd }}$ dimension by
-A(:,:,2) $=[56 ; 7$ 8]


## Matrixes and vectors command summary

- $x=[1,2,3]$, vector-row,
- $y=[1 ; 2 ; 3]$, vector-column,
- $x=0: 0.1: 0.8$, vector $x=[0,0.1,0.2,0.3 \ldots .0 .8]$,
- $A=[1,3,5 ; 5,6,7 ; 8,9,10]$, matrix,
- $A(1,2)$, element of matrix, 1. row, 2. column,
- $A(:, 2)$, second column of matrix,
- $A(1,:)$, first row of matrix ,
- $C=[A ;[10,20,30]]$ matrix with additional row,
- $A(:, 2)=[]$, deleting of second column,
- $B=A(2: 3,1: 2)$, part of matrix,
- $x$ ', transpose.


## Matrixes and vectors command summary ...

- size(A), matrix size,
- $\operatorname{det}(A)$, determinant,
- inv(A), inverse matrix,
- eye(3), unit matrix,
- zeros $(3,4)$, matrix of zeros,
- $\operatorname{rand}(3,5)$, matrix of random values,
- $\operatorname{sum}(A)$, sum of elements,
- $A^{*} x$, matrix-vector product (if dimensions are corresponding),
- $A .{ }^{*} B$, element multiplication of two matrixes.


## Thanks

## Questions ??

