Lecture Series - 2

Complex Numbers in MATLAB

by

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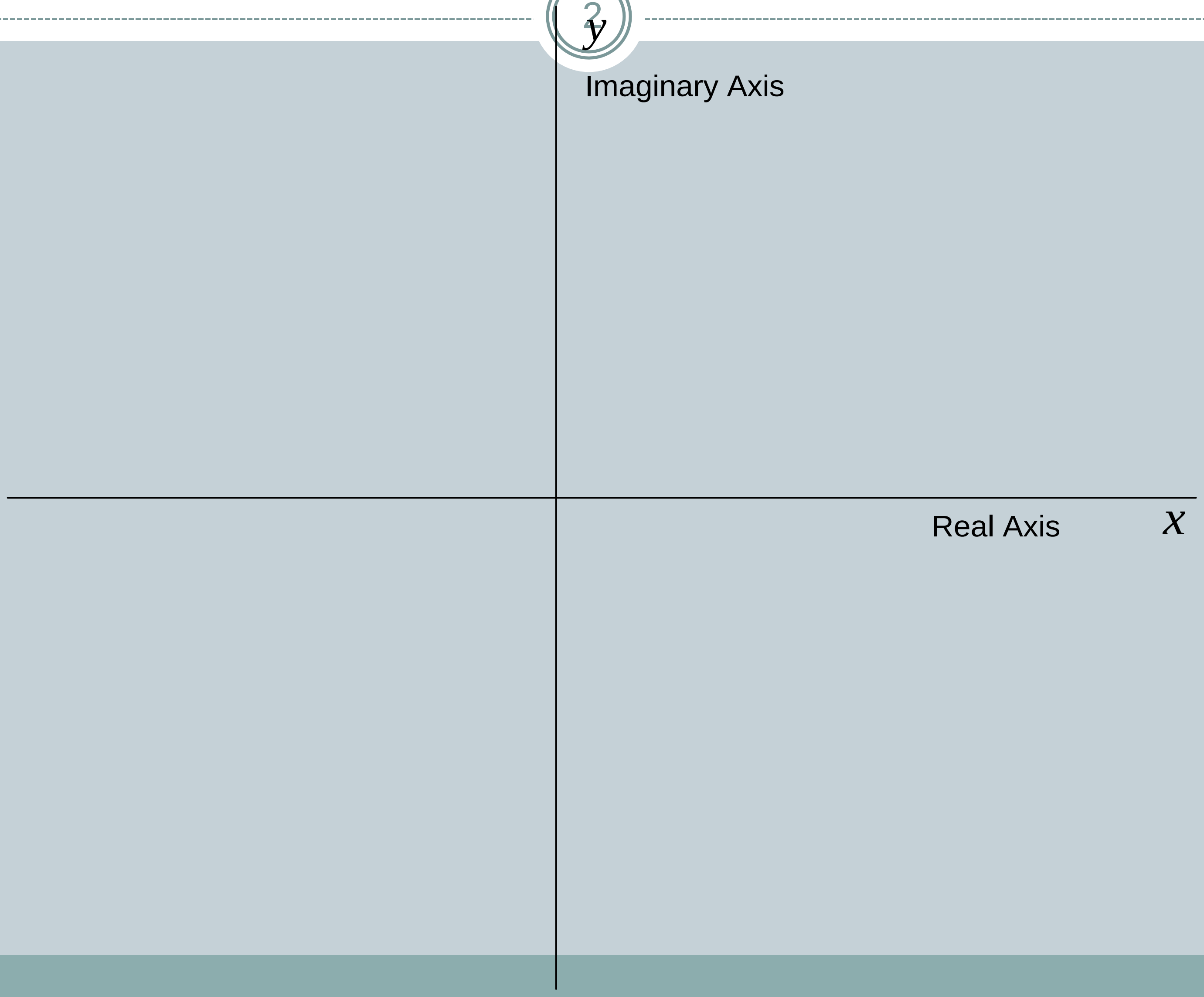
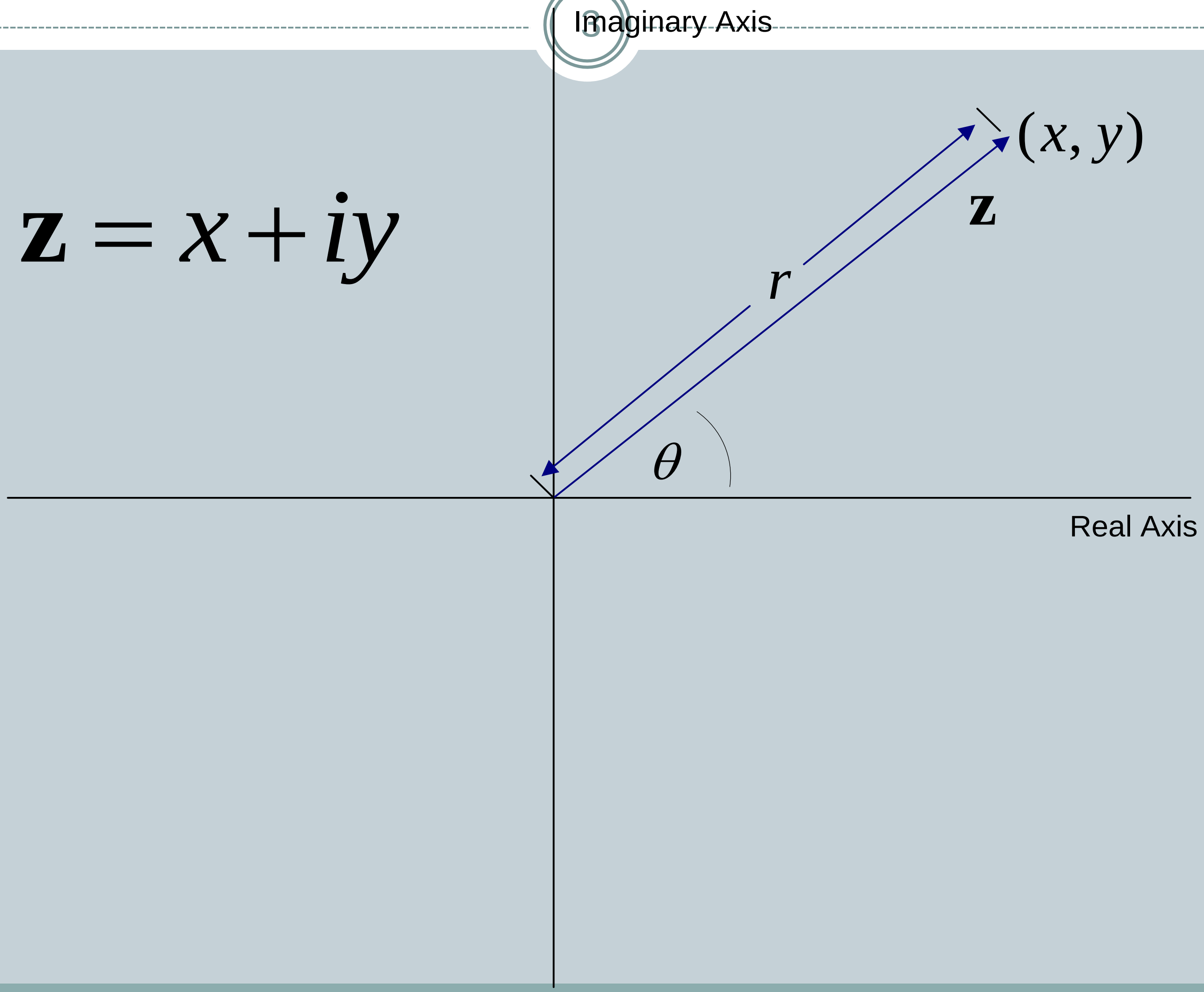
Complex Plane



Form of Complex Number

$$\mathbf{z} = x + iy$$



Conversion Between Forms

Polar to Rectangular:

$$x = r\cos\theta$$

$$y = r\sin\theta$$

Rectangular to Polar:

$$r = \sqrt{x^2 + y^2}$$

$$\theta = \text{ang}(\mathbf{z}) = \tan^{-1}\frac{y}{x}$$

Euler's Formula

$$e^{i\theta} = \cos\theta + i\sin\theta$$

$$\mathbf{z} = r\cos\theta + ir\sin\theta = r(\cos\theta + i\sin\theta)$$

$$\mathbf{z} = re^{i\theta}$$

Common Engineering Notation:

$$re^{i\theta} \square r\angle\theta$$

Convert the complex number to polar form:

$$\mathbf{z} = 4 + i3$$

$$r = \sqrt{x^2 + y^2} = \sqrt{(4)^2 + (3)^2} = 5$$

$$\theta = \tan^{-1}\frac{3}{4} = 36.87^\circ = 0.6435 \text{ rad}$$

$$\mathbf{z} = 5\angle 36.87^\circ \quad \text{or}$$

$$\mathbf{z} = 5e^{i0.6435}$$

Convert the complex number to polar form:

$$\mathbf{z} = -4 + i3$$

$$r = \sqrt{x^2 + y^2} = \sqrt{(-4)^2 + (3)^2} = 5$$

$$\theta = \tan^{-1}\left(\frac{3}{-4}\right) = 180^\circ - \tan^{-1}\frac{3}{4}$$

$$= 180^\circ - 36.87^\circ = 143.13^\circ = 2.498 \text{ rad}$$

$$\mathbf{z} = 5e^{i2.498}$$

Convert the complex number to rectangular form:

$$\mathbf{z} = 4e^{i2}$$

$$x = 4\cos 2 = -1.6646$$

$$y = 4\sin 2 = 3.6372$$

$$\mathbf{z} = -1.6646 + i3.6372$$

Addition of Two Complex Numbers

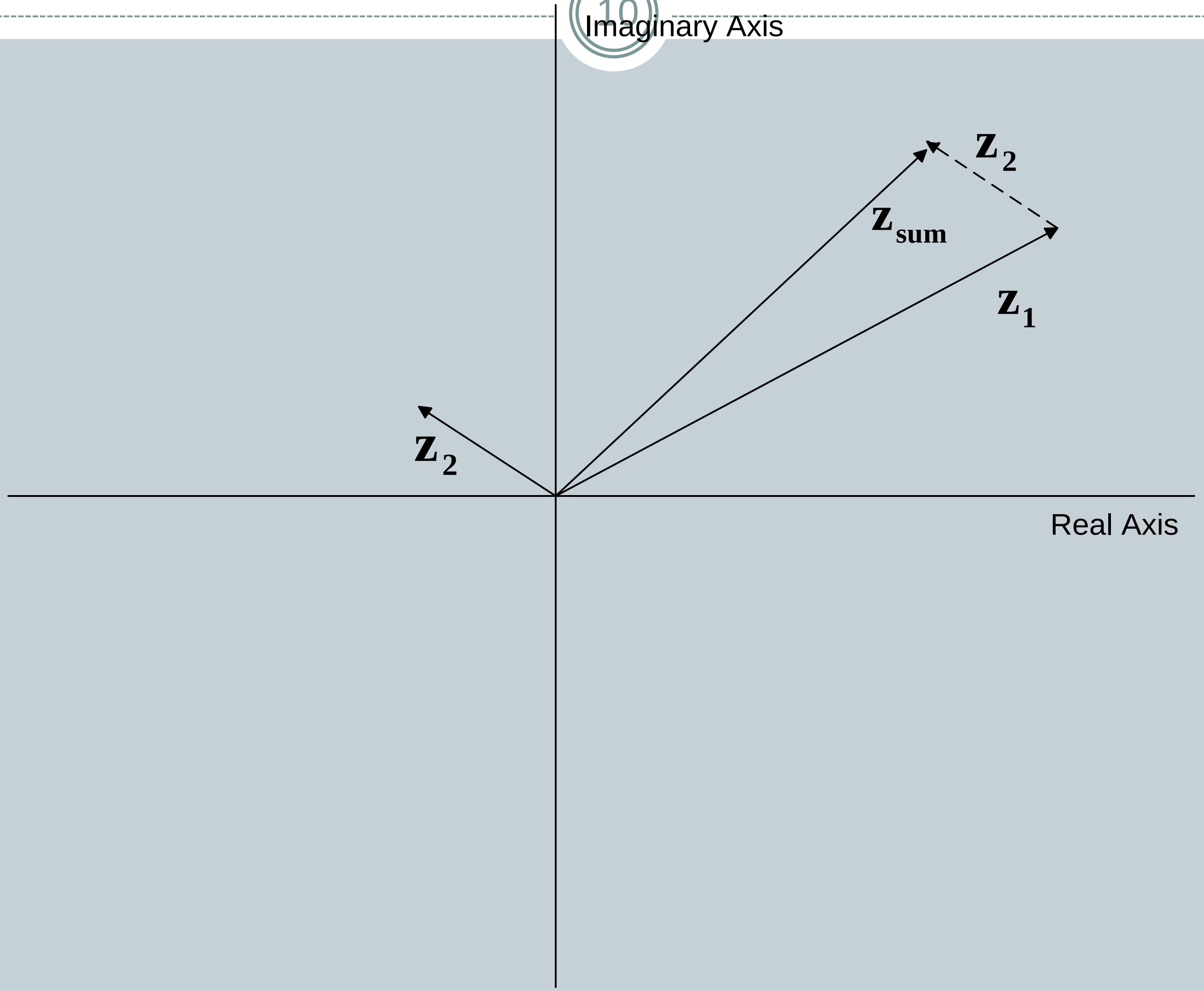
$$\mathbf{z_1} = x_1 + iy_1$$

$$\mathbf{z_2} = x_2 + iy_2$$

$$\begin{aligned}\mathbf{z_{sum}} &= \mathbf{z_1} + \mathbf{z_2} \\ &= x_1 + iy_1 + x_2 + iy_2 \\ &= x_1 + x_2 + i(y_1 + y_2)\end{aligned}$$

A geometric interpretation of addition is shown on the next slide.

Addition of Two Complex Numbers



Subtraction of Two Complex Numbers

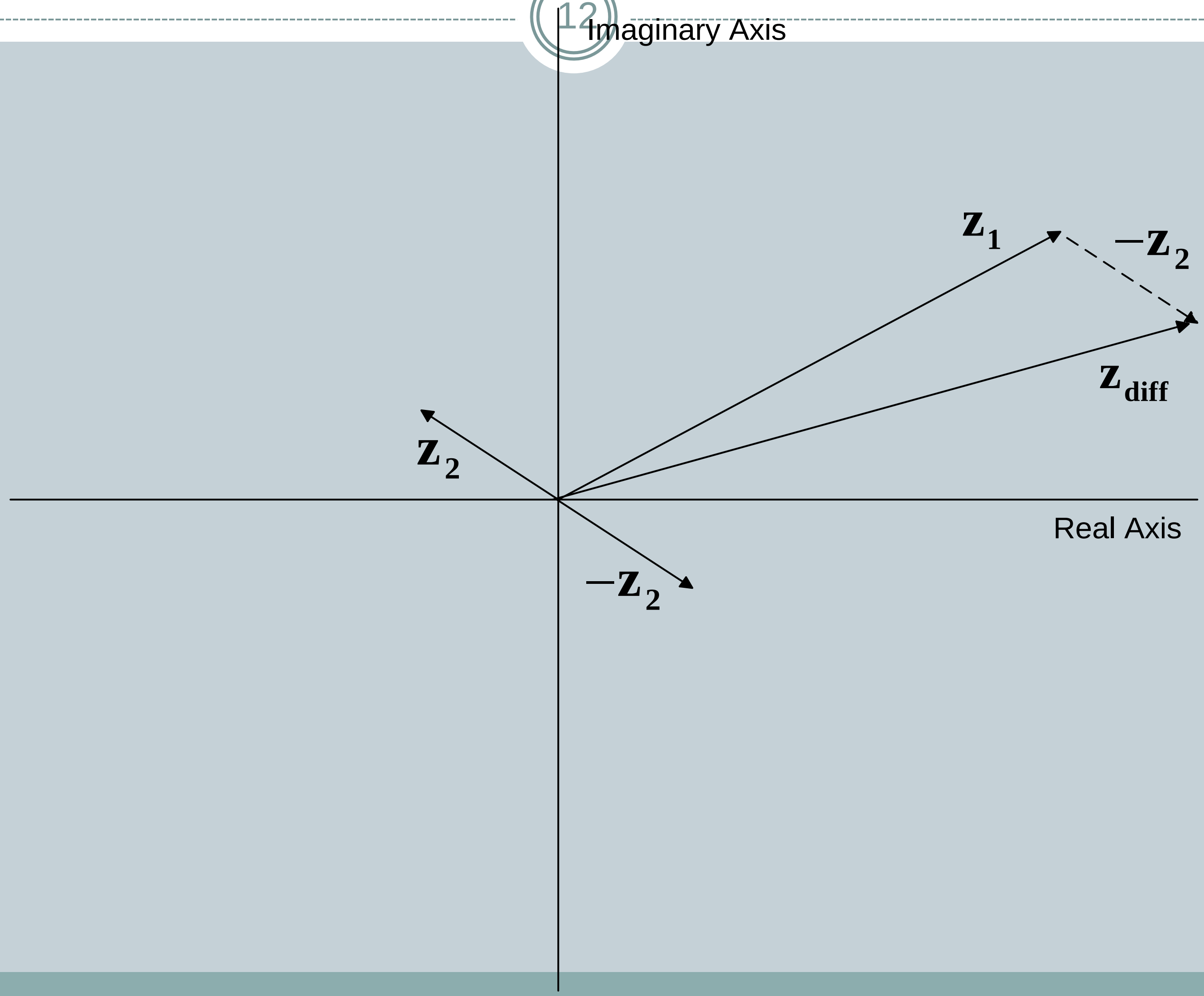
$$\mathbf{z_1} = x_1 + iy_1$$

$$\mathbf{z_2} = x_2 + iy_2$$

$$\begin{aligned}\mathbf{z_{diff}} &= \mathbf{z_1} - \mathbf{z_2} \\ &= x_1 + iy_1 - (x_2 + iy_2) \\ &= x_1 - x_2 + i(y_1 - y_2)\end{aligned}$$

A geometric interpretation of subtraction is shown on the next slide.

Subtraction of Two Complex Numbers



Multiplication in Polar Form

$$\mathbf{z_1} = r_1 e^{i\theta_1}$$

$$\mathbf{z_2} = r_2 e^{i\theta_2}$$

$$\mathbf{z_{prod} = z_1 z_2}$$

$$= \left(r_1 e^{i\theta_1}\right)\left(r_2 e^{i\theta_2}\right)$$

$$= r_1 r_2 e^{i(\theta_1 + \theta_2)}$$

Division in Polar Form

$$\mathbf{z_1} = r_1 e^{i\theta_1}$$

$$\mathbf{z_2} = r_2 e^{i\theta_2}$$

$$\mathbf{z_{div}} = \frac{\mathbf{z_1}}{\mathbf{z_2}} = \frac{\left(r_1 e^{i\theta_1}\right)}{\left(r_2 e^{i\theta_2}\right)}$$

$$= \frac{r_1}{r_2} e^{i(\theta_1 - \theta_2)}$$

Multiplication in Rectangular Form

$$\mathbf{z_1} = x_1 + iy_1$$

$$\mathbf{z_2} = x_2 + iy_2$$

$$\mathbf{z_{prod}} = (x_1 + iy_1)(x_2 + iy_2)$$

$$= x_1 x_2 + ix_1 y_2 + ix_2 y_1 + i^2 y_1 y_2$$

$$\mathbf{z_{prod}} = x_1 x_2 - y_1 y_2 + i(x_1 y_2 + x_2 y_1)$$

Complex Conjugate

Start with

$$\mathbf{z} = x + iy = re^{i\theta}$$

The complex conjugate is

$$\bar{\mathbf{z}} = x - iy = re^{-i\theta}$$

The product of z and $\bar{z}$ is

$$(\mathbf{z})(\bar{\mathbf{z}}) = x^2 + y^2 = r^2$$

Division in Rectangular Form

$$\mathbf{z}_{\mathbf{div}} = \frac{\mathbf{z}_{\mathbf{1}}}{\mathbf{z}_{\mathbf{2}}} = \frac{x_1 + iy_1}{x_2 + iy_2}$$

$$\mathbf{z}_{\mathbf{div}} = \frac{(x_1 + iy_1)(x_2 - iy_2)}{(x_2 + iy_2)(x_2 - iy_2)}$$

$$= \frac{x_1x_2 + y_1y_2 + i(x_2y_1 - x_1y_2)}{x_2^2 + y_2^2}$$

$$= \frac{x_1x_2 + y_1y_2 + i(x_2y_1 - x_1y_2)}{r^2}$$

Exponentiation of Complex Numbers: Integer Power

$$\mathbf{z}_{\mathbf{power}} = (\mathbf{z})^N$$

$$\mathbf{z}_{\mathbf{power}} = (re^{i\theta})^N = r^N e^{iN\theta}$$

$$= r^N \cos N\theta + ir^N \sin N\theta$$

$$\cos N\theta = \mathrm{Re}(e^{iN\theta})$$

$$\sin N\theta = \mathrm{Im}(e^{iN\theta})$$

MATLAB Complex Operations

- Complex number

- Construct complex data from real and imaginary components

```
>> c = complex(a,b)

>> z = 3 + 4i
z =
   3.0000 + 4.0000i
>> z = 3 + 4j
z =
   3.0000 + 4.0000i
```

MATLAB Complex Number Operations: Entering in Polar Form


```
>> z = 5*exp(0.9273i)
z =
   3.0000 + 4.0000i
>> z = 5*exp((pi/180)*53.13i)
z =
   3.0000 + 4.0000i
```

This result indicates that polar to rectangular conversion occurs automatically upon entering the number in polar form.

Rectangular to Polar Conversion


```
>> z = 3 + 4i
z =
  3.0000 + 4.0000i
>> r = abs(z)
r =
    5
>> theta = angle(z)
theta =
   0.9273
```

Real and Imaginary and Conjugate


```
>>real(z)
ans =
    3
>> imag(z)
ans =
    4
>> z1 = conj(z)
z1 =
   3.0000 - 4.0000i
```

Complex Algebra

- $Z_1 = 3+4i$
- $Z_2 = 2-5i$

- $Z_3 = Z_1+Z_2$
- $Z_4 = Z_1-Z_2$
- $Z_5 = Z_1*Z_2$
- $Z_6 = Z_1/Z_2$

Plotting complex number

- Use simple 'plot' function
- Plot (real, imaginary)
- Use 'compass' function
- Compass (z)

Exercise

- a= 3+2i b= 4+5i
- Find
  - Magnitude of a
  - Angle of a
  - Real part of a
  - Imaginary part of a
  - Conjugate of a
  - Plot a and b using 'plot' and 'compass'
  - a+b, a-b, a/b, a*b, a^2

Thanks

Questions ??